APPENDIX: "LICENSING AND THE INFORMAL SECTOR IN RENTAL HOUSING MARKETS: THEORY AND EVIDENCE"

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# 1 Proofs for Propositions

**Assumption 1** ensures that both markets exist in equilibrium. This amounts to the following requirements.

a. The rent differential is positive. The rent differential is positive iff

$$(1-\mu)(\mathcal{C}-\mathcal{F})+K\Theta(1-\alpha)-k\mathcal{U}\mu(1-\alpha)\geq 0.$$

b. Supply of high-quality housing: Substituting the equilibrium values of rents into $S^*(\lambda)>0$ yields

$$K(1-\alpha)(\mathcal{F}+\mathcal{X}\Delta\lambda(\Theta-\mathcal{U}\mu)-\mathcal{C})\geq 0.$$

c. Supply of low-quality housing: Substituting the equilibrium values of rents (from Proposition 1) into $S^*(\underline{\lambda})\geq 0$ yields the condition:

$$K\mathcal{X}(\underline{\lambda}-\alpha\overline{\lambda})(\Theta-\mathcal{U}\mu)+\mathcal{C}(K\alpha+\mathcal{X}\underline{\lambda}(1-\mu))+(K(1-\alpha)+\mathcal{X}\Delta\lambda(1-\mu))\Phi-\mathcal{F}(K+\mathcal{X}\underline{\lambda}(1-\mu))\geq 0.$$

d. Finally, $D^*(\overline{\lambda})\geq 0$ if

$$(1-\mu)(\mathcal{F}+\mathcal{X}\Theta\Delta\lambda)-K\mathcal{U}\mu(1-\alpha)\geq 0.$$

Since the system is fully determined when three of the four conditions are met, these conditions are also sufficient to ensure that $D^*(\underline{\lambda})\geq$ in equilibrium.

**Proposition 1** *Under Assumption 1, Equations 5, 7, 8, and 9 form the system of equations that yield the equilibrium $\overline{r}^*$ and $\underline{r}^*$. In this equilibrium, rents in the high and low-quality markets are given by*

$$\underline{r}^*=\frac{\underline{\lambda}(\mathcal{F}(1-\mu)+K\alpha(\Theta-U\mu)-\Phi(1-\mu))}{\alpha K+\underline{\lambda}\mathcal{X}(1-\mu)} \tag{10}$$

$$\overline{r}^*=\underline{r}^*+\frac{\Delta\lambda(k\Theta(1-\alpha)+(1-\mu)(\mathcal{C}-\mathcal{F})-K\mathcal{U}\mu(1-\alpha))}{K(1-\alpha)+\mathcal{X}\Delta\lambda(1-\mu)}. \tag{11}$$

**Proof.** Setting Equation (8) equal to (5) and Equation (9) equal to (7) yields a system of linear equations in rents which, when solved, yield the equilibrium rents in this proposition. ■

**Proposition 2** *An increase in the expected regulated sector fine $f$*

1. *decreases the rent differential between formal and informal/low-quality housing*
2. *increases the rent of low-quality housing*
3. *has an ambiguous impact on rent of high-quality housing*
4. *decreases the quantity of informal housing*
5. *increases the quantity of high-quality housing*
6. *increases the vacancy rate*
7. *increases the level of homelessness.*

**Proof.**

1. $\frac{\partial(\bar{r}^* - \underline{r}^*)}{\partial f} = -\frac{\Delta\lambda(1-\mu)}{K(1-\alpha)+\mathcal{X}\Delta\lambda(1-\mu)} < 0.$
2. $\frac{\partial \underline{r}^*}{\partial f} = \frac{\underline{\lambda}(1-\mu)}{\alpha K + \underline{\lambda}\mathcal{X}(1-\mu)} \frac{\partial F}{\partial f} > 0.$
3. $\frac{\partial \bar{r}^*}{\partial f} = \frac{k(1-\mu)(\underline{\lambda} - \bar{\lambda}\alpha)}{(\alpha k + \underline{\lambda}\mathcal{X}(1-\mu))(k(1-\alpha)+\mathcal{X}\Delta\lambda(1-\mu))} \frac{\partial F}{\partial f} \lesseqgtr 0.$
4. Observe that the rent differential is decreasing in $f$ and $\underline{r}^*$ is increasing in $f$. Hence $D^*(\underline{\lambda})$ is decreasing in $f$.
5. $\frac{\partial D^*(\bar{\lambda})}{\partial f} = -\frac{\partial(\bar{r}^* - \underline{r}^*)}{\partial f} > 0.$
6. The vacancy rate is:

$$V \equiv \frac{1}{K}\left(K\sum_{d=1}^{M} l(d)d - \frac{1}{\alpha}\Phi - \frac{\underline{r}}{\alpha}\mathcal{X} + \frac{\mathcal{F}}{\alpha}\right).$$

Thus,

$$\frac{\partial V}{\partial f} = -\frac{\mathcal{X}}{\alpha}\frac{\partial \underline{r}}{\partial f} + \frac{\partial F}{\partial f}\frac{1}{\alpha},$$

which simplifies to

$$\frac{1}{\alpha K}\frac{\partial F}{\partial f}\frac{\alpha K}{\alpha K + \underline{\lambda}(1-\mu)\mathcal{X}} > 0.$$

7. The level of homelessness is $\frac{\underline{r}}{\underline{\lambda}}$. Since $\underline{r}^*$ rises with $f$, homelessness rises.

■

**Proposition 3** *An increase in the regulatory threshold*

1. *has an ambiguous effect on the rent differential between formal and informal/low-quality housing if $f > f_N$. Otherwise, decreases the rent differential*
2. *decreases the rent of low-quality housing if and only if $f > f_N$*
3. *has an ambiguous impact on rent of high-quality housing*
4. *has an ambiguous effect on the quantity of informal/low-quality housing if $f > f_N$. If $f < f_N$, then it decreases the quantity of informal/low-quality housing*
5. *has an ambiguous effect on the quantity of high-quality housing*
6. *reduces vacancies if and only if $f > f_N$*
7. *reduces the level of homelessness if and only if $f > f_N$*

**Proof.** First, we wish to make a few observations concerning $\mathcal{C}$ and $\mathcal{F}$. Define $\Delta$ as the difference operator with respect to the limits of the summation. That is,

$$\Delta(\mathcal{C}) = c\sum_{d^P+1}^{M} dl(d) - c\sum_{d^P}^{M} dl(d).$$

Note that by construction, for any $d^P$,

$$c\sum_{d^P+1}^{M} dl(d) = c\sum_{d^P}^{M} l(d)d - c\left(l(d^P)d^P\right) = \mathcal{C} - c\left(l(d^P)d^P\right).$$

Since the second term is strictly negative, it follows that

$$c\sum_{d^P+1}^{M} dl(d) < c\sum_{d^P}^{M} dl(d)$$

for $\forall d^P$. Hence, $\Delta(\mathcal{C}) < 0 \ \forall d^P$.

Similarly, let

$$\Delta(\mathcal{F}) = \mathcal{F}(d^P+1) - \mathcal{F}(d^P).$$

Then by construction,

$$\mathcal{F}(d^P+1) = f_N\sum_{d=1}^{d^P+1} l(d)d^3 + f\sum_{d^P+2}^{M} l(d)d^3$$

$$= f_N \left( \sum_{d=1}^{d^P} l(d)d^3 + l(d^P+1)(d^P+1)^3 \right) + f \left( \sum_{d^P+1}^{M} l(d)d^3 - l(d^P+1)(d^P+1)^3 \right)$$

$$= f_N \sum_{d=1}^{d^P} l(d)d^3 + f \sum_{d^P+1}^{M} l(d)d^3 + (f_N - f) l(d^P+1)(d^P+1)^3.$$

The last term in the previous expression is strictly negative iff $f > f_N$. Hence, for any $d^P$ it follows that

$$\Delta(\mathcal{F}) < 0$$

if and only if $f > f_N$.

1. An increase in the regulatory thresholds changes the rent differential by

$$\frac{\partial(\overline{r} - \underline{r})}{\partial d^P} = [\Delta\mathcal{C} - \Delta\mathcal{F}] \frac{\Delta\lambda(1-\mu)}{K(1-\alpha) + \mathcal{X}\Delta\lambda(1-\mu)}.$$

   This expression may be positive or negative when $f > f_N$ and it is negative when $f < f_N$.

2. An increase in the regulatory thresholds changes the rent of low-quality housing by

$$\frac{\partial \underline{r}^*}{\partial d^P} \approx \frac{\underline{\lambda}(1-\mu)}{\alpha K + \underline{\lambda}\mathcal{X}(1-\mu)} (\Delta(\mathcal{F})) < 0$$

   if and only if $f > f_N$.

3. An increase in the regulatory thresholds changes the rent of high-quality housing by

$$\frac{\partial \overline{r}^*}{\partial d^P} \approx (\Delta(\mathcal{F})) \left( \frac{K(\underline{\lambda} - \overline{\lambda}\alpha)(1-\mu)}{(\alpha K + \underline{\lambda}\mathcal{X}(1-\mu))(K(1-\alpha) + \mathcal{X}\Delta\lambda(1-\mu))} \right) + (\Delta(\mathcal{C})) \frac{\Delta\lambda(1-\mu)}{K(1-\alpha) + \mathcal{X}\Delta\lambda(1-\mu)}. \quad (12)$$

   This expression can be positive or negative depending on whether $(\underline{\lambda} - \overline{\lambda}\alpha)$ and $f \lesseqgtr f_N$.

4. An increase in the regulatory thresholds changes the quantity of low-quality housing by

$$\frac{\partial D^*(\underline{\lambda})}{\partial d^P} \approx -\Delta(\mathcal{F}) \left( \frac{(K + \overline{\lambda}\mathcal{X}(1-\mu))(1-\mu)}{(K(1-\alpha) + \mathcal{X}\Delta\lambda(1-\mu))(\alpha K + \underline{\lambda}\mathcal{X}(1-\mu))} \right) + \Delta(\mathcal{C}) \left( \frac{1-\mu}{K(1-\alpha) + \mathcal{X}\Delta\lambda(1-\mu)} \right). \quad (13)$$

The second term is negative because $\Delta(\mathcal{C}) < 0$. The first term is positive when $f > f_N$. Hence, the effect is ambiguous if $f > f_N$. If $f < f_N$, then both terms are negative so $D^*(\underline{\lambda})$ is decreasing in $d^P$.

5. Since $d^P$ has an ambiguous effect on the rent differential when $f > f_N$, the quantity of high-quality housing may rise or fall with $d^P$. When $f < f_N$, then the rent differential falls with $d^P$ such that $D^*(\overline{\lambda})$ increases with $d^P$.

6. From the expression for vacancies,

$$\Delta(F)\frac{1}{\alpha K}\frac{\partial F}{\partial f}\frac{\alpha K}{\alpha K + \underline{\lambda}(1-\mu)\mathcal{X}} > 0.$$

   When $f > f_N$ ($f < f_N$), then $\mathcal{F}$ is decreasing (increasing) in $d^P$. Hence, vacancies fall (rise) with the threshold.

7. Since homelessness is given by $\frac{r^*}{\underline{\lambda}}$, as low-quality rents fall, homelessness also falls if $f > f_N$.

■

# 2 High-Quality Estimates for One and Two Dwelling Properties

In order to impute the quality status of units in the unregulated sector we take the following steps:

1. First, we limit our sample to only properties with three dwelling units, which are presumably the most similar to one and two dwelling units in the unregulated sector. After excluding observations that do not have a full set of covariates, we are left with 9,177 observations. Descriptive statistics for the sample can be found in Table 1.

2. We then run a probit model with the dependent variables equal to one when licensed, and zero otherwise, against citations per dwelling unit, log of market value per unit, year built, rowhouse and semidetached dummy variables, and city ward fixed effects. To guard against endogeneity between licensing status and citations and market value, we lag these two variables one year.

3. Using the results of the probit model (Table 2), we impute the probability of being licensed for one and two dwelling properties.

Table 1: Descriptive Statistics for High Quality Extrapolation Model

| | Mean | Std. Dev. |
|---|---|---|
| licensed | 0.55 | 0.50 |
| totalmarketvalue_perunit | 50863.18 | 39206.11 |
| citations_perunit | 0.25 | 0.55 |
| yearbuilt | 1911.11 | 16.32 |
| rowhouse | 0.02 | 0.13 |
| semidetached | 0.00 | 0.04 |
| $N$ | 9,177 | |

Table 2: Extrapolation Model Results

| | Estimate | t-Statistic |
|---|---|---|
| Lagged Log totalmarketvalue_perunit | 0.345*** | (21.785) |
| Lagged citations_perunit | -0.196*** | (-7.708) |
| yearbuilt | 0.002* | (2.097) |
| rowhouse | -0.911*** | (-7.563) |
| semidetached | 0.645 | (1.487) |
| Constant | -7.416*** | (-3.687) |
| $N$ | 9,177 | |

Note: Regression includes ward fixed effects.

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$